Color Neutral Ground State of 2SC Quark Matter

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We construct a new color neutral ground state of two-flavor color superconducting quark matter. It is shown that, in contrast with the conventionally considered ground state with diquark pairing in only one color direction, this new state is stable against arbitrary diquark fluctuations. In addition, the thermodynamical potential is found to be lower for this new state than for the conventional one.

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Recent investigations on the QCD phase diagram have discovered a rich diversity of color superconducting quark matter phases at low temperatures and intermediate densities [1]. Particularly interesting are possible implications of these results for the physics of compact stars [2] as well as heavy ion collision experiments [3] addressing the domain of densities and temperatures where the strange quarks are still heavy and confined [4, 5] . In both systems, the constraint of global color neutrality has to be fulfilled. In addition, the constraint of global electric neutrality has to be satisfied when macroscopic objects like compact stars are considered and flavor changing processes have enough time to adjust u and d quark chemical potentials according to β-equilibrium.

In view of the nonperturbative character of QCD, the theoretical treatment of hadronic matter at the vicinity of the phase transitions for low temperatures and finite densities is a problem of highest complexity, where rigorous theoretical approaches are not yet available and lattice QCD simulations are up to now not applicable. Therefore one has to rely on effective field-theoretical models of interacting quark matter, which are built taking into account the symmetry requirements of the QCD lagrangian and offer the possibility of dealing with the yet simplified interactions in a systematic way. Chiral quark models of QCD that adopt a current-current form of the interaction with mesonic and diquark components have been particularly useful, since the theories can be bosonized in a straightforward way. In the meson-diquark representation of these models an effective quantum hadrodynamics can be derived [6], but it has not yet been used to explore the QCD phase diagram. A preparatory step for this formidable task is the investigation of the mean field approximation (MFA), where important progress has been recently made within a nonlocal, covariant formuation that has been extended even to the study of color superconductivity in the QCD phase diagram [7].

The two-flavor color superconductivity (2SC) phase of quark matter has been first considered in instanton-motivated QCD models in [8], where the question of color neutrality was disregarded. However, it was soon realized that the 2SC phase in which color symmetry is broken by the orientation of the diquark field in one of the color directions (2SC-b) entails a mismatch in the quark densities of paired and unpaired colors provided that color chemical potentials are all equal to each other, $\mu_r = \mu_g = \mu_b$. Therefore, in the 2SC-b state color neutrality requires color charge chemical potentials to be readjusted so that $\mu_8 = \frac{1}{2}(\mu_r + \mu_g - 2\mu_b)$ acquires a nonvanishing value while $\mu_3 = \frac{3}{2}(\mu_r - \mu_g)$ remains zero due to the degeneracy of the red and green colors. While this adjustment of $\mu_8 \neq 0$ has long been considered a proper solution of the color neutrality constraint [9], a recent investigation of fluctuations around the mean field oriented in the blue (0,0,1) direction has revealed the instability of this state once color neutrality is required [10]. In the present paper we investigate the entire space of mean-field orientations in order to look for color neutral states which are stable against fluctuations. We find that these correspond to color neutral symmetric states (2SC-s) for which the condensates are equal in modulus in all the three directions of the color space.

As in Ref. [10], we consider the simplest version of the flavor SU(2) Nambu$-$Jona-Lasinio model [11, 12, 13], extended so as to include the quark-quark interaction sector and finite chemical potentials

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu + \mu\gamma_0 - \hat{m}\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] + G_D\left(\bar{\psi}^c_{i\alpha}i\gamma^5\epsilon^{ij}\epsilon^{\alpha\beta\gamma}\psi_{j\beta}\right)\left(\bar{\psi}_{i\alpha}i\gamma^5\epsilon^{ij}\epsilon^{\alpha\beta\gamma}\psi^c_{j\beta}\right) \quad . \qquad (1)$$

Here $\hat{m}$ is the diagonal current mass matrix for light quarks, G_S and G_D are coupling constants in color singlet and anti-triplet channels respectively, and $\psi_{i\alpha}$ stands for quark fields with flavor index $i = u, d$ and color index $\alpha = r, g, b$ (charge conjugated fields are given by $\psi^c_{i\alpha} = i\gamma^2\gamma^0\bar{\psi}^T_{i\alpha}$). The Pauli matrices $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ act in flavor space, while and ϵ^{ij} and $\epsilon^{\alpha\beta\gamma}$ are totally antisymmetric tensors in flavor and color spaces, respectively. In the present letter we are mainly concerned with the effect of color neutrality on the ground state of a two-flavor

color superconductor, therefore we will restrict here the discussion to the flavor symmetric case and consider the extension to electrically neutral matter elsewhere. Regarding quark chemical potentials, the elements of the matrix

$$\mu = \mathrm{diag}\,(\mu_r, \mu_g, \mu_b, \mu_r, \mu_g, \mu_b) \qquad (2)$$

can be written as

$$\begin{aligned} \mu_r &= \mu_B/3 + \mu_8/3 + \mu_3/3 \ , \\ \mu_g &= \mu_B/3 + \mu_8/3 - \mu_3/3 \ , \\ \mu_b &= \mu_B/3 - 2\mu_8/3 \ , \end{aligned} \qquad (3)$$

where μ_B is the baryon chemical potential, while μ_8 and μ_3 are introduced to ensure color charge neutrality. Our aim is to discuss the color superconducting phase of the model in the mean-field approximation, which in general is characterized by nonvanishing diquark condensates

$$\Delta_\alpha = -\,2\,G_D\,\langle \bar{\psi}^c_{i\beta} i\gamma^5 \epsilon^{ij} \epsilon^{\beta\gamma\alpha} \psi_{j\gamma} \rangle, \quad \alpha = r, g, b \ . \qquad (4)$$

For standard values of the diquark coupling, $G_D = \frac{3}{4} G_S$, there is no simultaneous chiral symmetry breaking in this phase. In addition, since for light quarks the current quark masses are significantly smaller than the typical values of μ_B and Δ_α, for the purpose of the present study we can safely neglect both these small masses and the corresponding mesonic mean fields. Within this limit, we proceed to calculate the thermodynamical potential per unit volume at zero temperature. For convenience we perform our calculations in Euclidean space, where the thermodynamical potential in MFA is given by

$$\Omega^{MFA} \;=\; \frac{\Delta^2}{4G_D} - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\ \ln\ \det M \ , \qquad (5)$$

and the constraint of global color neutrality has to be obeyed, i.e. color charge densities should vanish

$$Q_\alpha = -\partial\Omega/\partial\mu_\alpha = 0 \ , \quad \alpha = 3, 8 \ . \qquad (6)$$

Here we have defined $\Delta^2 = \sum_{\alpha=r,g,b} \Delta_\alpha^2$, and the space integral is regulated as usual by introducing a sharp three-momentum cutoff Λ. The inverse fermion propagator M is a 48×48 matrix in Nambu-Gorkov, Dirac, color and flavor spaces, and can be conveniently written as

$$M = \begin{pmatrix} M^+ & 0 \\ 0 & M^- \end{pmatrix} \ , \qquad M^- = -{M^+}^\dagger \ , \qquad (7)$$

with

$$M^+ \;=\; \begin{pmatrix} (G^+_{0r})^{-1} & 0 & 0 & 0 & -\Delta'_b & \Delta'_g \\ 0 & (G^+_{0g})^{-1} & 0 & \Delta'_b & 0 & -\Delta'_r \\ 0 & 0 & (G^+_{0b})^{-1} & -\Delta'_g & \Delta'_r & 0 \\ 0 & \Delta'_b & -\Delta'_g & (G^-_{0r})^{-1} & 0 & 0 \\ -\Delta'_b & 0 & \Delta'_r & 0 & (G^-_{0g})^{-1} & 0 \\ \Delta'_g & -\Delta'_r & 0 & 0 & 0 & (G^-_{0b})^{-1} \end{pmatrix} \ , \qquad (8)$$

where $\Delta'_\alpha = i\gamma_5\Delta_\alpha$ and $(G^\pm_{0\alpha})^{-1} = -[(p_4 \mp i\mu_\alpha)\gamma_4 + \vec{p}\cdot\vec{\gamma}]$ are 4×4 matrices in Dirac space. After some algebra, it is seen that the determinant can be cast into the form

$$\det\,M = (\ S^+\ S^-\ )^4 \ , \qquad (9)$$

where

$$\begin{aligned} S^\pm \;=\; & |C^\pm_r C^\pm_g C^\pm_b|^2 + |C^\pm_r \Delta_r^2 + C^\pm_g \Delta_g^2 + C^\pm_b \Delta_b^2|^2 \\ & + 2\Big[|C^\pm_r|^2\,\mathrm{Re}\,({C^\pm_g}^* C^\pm_b)\ \Delta_r^2 + \mathrm{cycl.perm.}\{rgb\}\Big] \end{aligned} \qquad (10)$$

with

$$C^\pm_\alpha = \mu_\alpha\ \pm\ |\vec{p}|\ +\ i\,p_4 \ .$$

With this general expression at hand we can investigate the problem of finding the most favored state under the constraint of the color neutrality, i.e. $Q_3 = Q_8 = 0$.

Since the expression in Eq.(10) is totally symmetric under cyclic permutations of the three color indices, a cubic symmetry is expected in the three-dimensional space spanned by the three color directions along which the magnitudes of the colored diquark condensates are the coordinates. In the standard Cartesian representation, the condensate vector in color space is given by

$$\vec{\Delta} = \Delta_r\ \vec{e}_r + \Delta_g\ \vec{e}_g + \Delta_b\ \vec{e}_b \ . \qquad (11)$$

Due to the above mentioned symmetry, in what follows we will restrict our discussion to the sector defined by nonnegative values of the Cartesian coordinates Δ_α. In this representation, the 2SC-b state is given by $\vec{\Delta}_{\mathrm{2SC}-b} = (0, 0, \Delta_b)$, whereas the color symmetric state is $\vec{\Delta}_{\mathrm{2SC-s}} = (\Delta_s, \Delta_s, \Delta_s)$, i.e. a vector pointing from the center to one of the edges of a cube in color space. For the 2SC-s state color neutrality is achieved with $\mu_8 = \mu_3 = 0$, while

everywhere else color symmetry is broken, entailing that either μ_8, μ_3 or both have to be different from zero in order to fulfill the color neutrality constraint.

The observation made in [10] stating that a 2SC-b state defines a saddle point of the thermodynamical potential (5) in the order parameter space, being a minimum in the blue direction but a maximum in the red and green ones, leads to the important fact that the 2SC-b state widely considered in the literature is not the true ground state of quark matter in the 2SC phase. Now the problem is to find the true 2SC ground state, which should be thermodynamically more favorable not only by a lower energy but also by its stability with respect to fluctuations in the amplitude and the orientation of the condensate. We show here that the 2SC-s state proposed in this paper fulfills these conditions. To do this, we perform a numerical analysis taking a phenomenologically acceptable set of model parameters, namely $\Lambda = 653$ MeV, $G_S\Lambda^2 = 2.14$ [13], together with $G_D = \frac{3}{4}G_S$. First, we show that the difference between the thermodynamical potentials of the 2SC-b and 2SC-s states is positive along the relevant range of baryochemical potentials, say 350 MeV $\leq \mu_B/3 \leq 600$ MeV. The corresponding curve is plotted in the upper panel of Fig. 1. Second, in the lower panel of Fig. 1 we prove the stability of the 2SC-s solution by showing the strict positivity of the eigenvalues k_1, k_2, k_3 of the curvature tensor

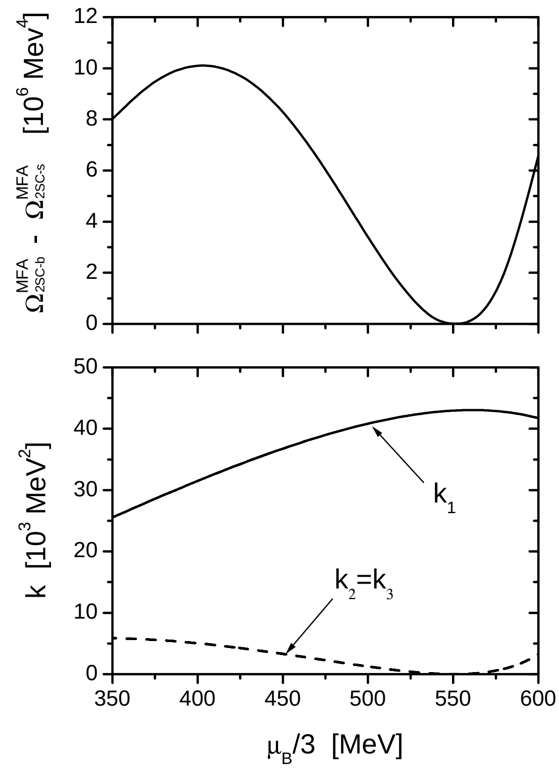

FIG. 1: Upper panel: Difference between the thermodynamical potentials of two color neutral states, namely the standard one –with broken color symmetry– and the new color symmetric one, as functions of the baryon chemical potential μ_B at temperature $T = 0$. Lower panel: Eigenvalues k_1, $k_2 = k_3$ of the curvature tensor obtained from the thermodynamical potential at the color-symmetric minimum. Strict positivity proves the stability w.r.t. arbitrary small amplitude fluctuations. Color neutrality has been imposed, but not electric charge neutrality.

$$K_{ij} = \frac{1}{2}\frac{\partial\Omega^{MFA}}{\partial\Delta_i\partial\Delta_j}\bigg|_{\mathrm{2SC-s}} \tag{12}$$

derived from the thermodynamical potential in the 2SC-s state. Interestingly, the largest eigenvalue k_1 corresponds to fluctuations in the "radial" direction $(1, 1, 1)$, while the two eigenvalues denoting the curvature in the orthogonal "angular" directions are found to be degenerate ($k_2 = k_3$).

Note that a special situation occurs at a chemical potential $\mu_B/3 \simeq 550$ MeV, where the 2SC-s state becomes energetically degenerate with the 2SC-b state and simultaneously the curvature in the angular directions vanishes. This implies that all orientations of the condensate vector $\vec{\Delta}$ are equivalent to each other, i.e. the cubic symmetry degenerates to a spherical one and the preference of the 2SC-s state gets lost (one has in this case $\Delta_b = \sqrt{3}\Delta_s$). In contrast, for all other values of μ_B the 2SC-s state is preferred, owing to the penalty introduced for all other states which need finite color chemical potentials to achieve color neutrality.

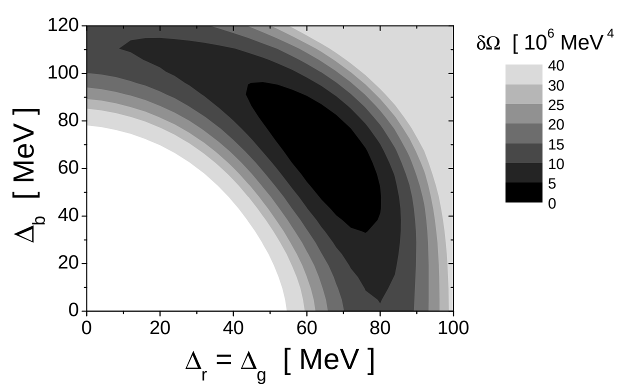

FIG. 2: The penalty for the thermodynamical potential, $\delta\Omega = \Omega^{MFA} - \Omega^{MFA}_{2SC-s}$, induced by the color neutrality constraint in the plane of the order parameters Δ_b and $\Delta_r = \Delta_g$ for $\mu_B = 1200$ MeV. Since $\mu_3 = 0$, the penalty is a function of μ_8 and both vanish in the 2SC-s state where $\Delta_r = \Delta_g = \Delta_b$.

The behavior of this penalty $\delta\Omega$ in the condensate state space is illustrated by the contour plot shown in Fig. 2 for a baryochemical potential $\mu_B = 1200$ MeV chosen such that a maximal effect can be demonstrated, see Fig. 1. In this plot we consider for simplicity the plane spanned by the axes $\Delta_r = \Delta_g$ and Δ_b, so that red and green colors are degenerate and one has $\mu_3 = 0$. The penalty, arising from color symmetry breaking, is a function of μ_8 and vanish in the 2SC-s state for which $\Delta_r = \Delta_g = \Delta_b$ and $\mu_8 = 0$. This figure strongly suggests that the 2SC-s

state is, in fact, the absolute minimum of the mean field thermodynamical potential.

Another important feature of the 2SC-s state concerns the corresponding 12 quasiparticle modes. It turns out that for this state the dispersion relations and degeneracy factors are given by

$$E_0^{\pm} \; = \; |\vec{p}\,| \pm \mu_B/3 \qquad [\,\times 2\,]\,, \tag{13}$$
$$E_\Delta^{\pm} \; = \; \sqrt{(|\vec{p}\,| \pm \mu_B/3)^2 + \Delta^2} \qquad [\,\times 4\,]\,, \tag{14}$$

where $\Delta = \sqrt{3}\Delta_s$. This means that the system contains two gapless modes, just like in the case of the conventional 2SC-b state. However, in the present case the gapless modes cannot be identified with the original u and d quarks of "blue" (unpaired) color in the original $\{r, g, b\}$ color basis but arise as a combination of all three color states.

In summary, we have shown in this letter that in the case of color neutrality the ground state of 2SC quark matter should be constructed in a "democratic" way, so that color symmetry is not broken by the choice of the orientation of the condensate vector in color space. For this state, the condition of color neutrality is fulfilled in a trivial way, since the penalty induced by otherwise necessary color chemical potentials is avoided. We have shown that this 2SC-s ground state is stable against fluctuations, thus the problem observed in [10] is solved. The 2SC-s state can serve as a starting point for considering hadronic correlations on the superconducting QCD vacuum, where due to the entanglement of the quark color states in the new basis color neutral quark and diquark excitations arise along the radial direction besides colored excitations in the tangential plane. The conclusions drawn so far should not be qualitatively altered when the additional constraint of electrical neutrality is imposed, which is important for applications in compact stars. Flavor asymmetry induced by this constraint could possibly inhibit the formation of the 2SC state in regions of the neutron star matter phase diagram. However, provided that a phase transition to 2SC quark matter is accomplished, it should be described by the 2SC-s state introduced in this paper. This issue, together with other extensions of the present work, will be explicitly discussed in forthcoming publications.

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